



Electromagnetic Fields (EPM 211)

The exam consists of SIX questions in THREE pages.

Useful Relations

Constants

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ F/m}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

Laplacian

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\nabla^2 V = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial V}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\nabla^2 V = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial V}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial V}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 V}{\partial \phi^2}$$

PART (I) (60 Marks)

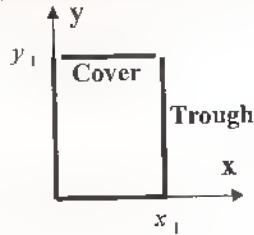
Question (1)

- a) State Gauss's law. Then using Gauss's law, find an expression for Maxwell's first equation.
 - b) The cylindrical surface $r = 0.3 \text{ m}$ contains a charge density of 40 C/m^2 , while that at $r = 0.5 \text{ m}$ contains -20 C/m^2 .
 - i) What charge density must exist on the surface $r = 0.4 \text{ m}$ so that the total charge is zero.
 - ii) Using these values of ρ_s , calculate D_r as a function of r and sketch D_r versus r for $0 < r < 0.7 \text{ m}$.
 - iii) If ρ_s is changed to -20 C/m^2 on the surface $r = 0.5 \text{ m}$, sketch D_r versus r .
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Question (2)

- a) A parallel plate capacitor is constructed using two very thin and rectangular plates of aluminum, of area S . A battery is used to charge this capacitor to a DC voltage of V_0 using air as a dielectric media initially. When the capacitor was fully charged, the battery was removed and a dielectric slab of dielectric constant ϵ_r was inserted. Assuming $S = 500 \text{ cm}^2$, $d = 6.0 \text{ cm}$, dielectric thickness = 0.25 cm , $\epsilon_r = 6.0$, and $V_0 = 80 \text{ volt}$. **Calculate:**
- The capacitance C_0 before the slab is introduced and the free charge.
 - The electric field intensity E in the air and in the dielectric.
 - The potential difference between the plates and the capacitance with the slab in place.
- b) A long rectangular metal trough with insulated cover is shown in cross section in Fig. Q2.b. The trough is at zero potential, and the cover is at potential V_1 . **Show** that the potential at any point (x, y) inside is given by

$$V = \sum_{n \text{ odd}} \frac{4 V_1 \sin \frac{n \pi x}{x_1} \sinh \frac{n \pi y}{x_1}}{n \pi \sinh \frac{n \pi y_1}{x_1}}$$



Question (3)

Given a rectangular current loop, as shown in Fig. Q3, the value of current I equals 50 Amp.

- Determine** the torque developed on the current loop when a magnetic field density $\underline{B} = 0.5 \underline{a}_x + 0.2 \underline{a}_z \text{ T}$, (in X and Z direction), is applied to the current loop.
- Sketch** the final position of the current loop if it is free to rotate.
- What** will happen if the magnetic field density \underline{B} is changed to be equal $0.5 \underline{a}_x$.
- Explain** how you could use this idea to understand the basic theory of the electrical motor.

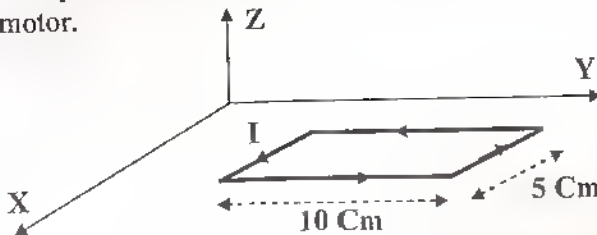


Fig. Q3



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Question (1)

Two concentric cylindrical conductors of radius $a = 0.02$ m and $b = 0.05$ m. The inner cylinder has a charge density $\rho_{sa} = 40$ nC/m² while the outer cylinder has ρ_{sb} such that the electric field exists between the two cylinders but they are zero elsewhere.

- Find the surface charge density ρ_{sb} ,
 - Derive expressions for the electric flux density and field strength in all regions,
 - Find the potential difference between the two cylinders,
 - Find the energy stored in the system per unit length,
 - Draw the distribution of the electric field intensity versus r in all regions if uniform line charge of density $\rho_L = 100$ nC/m is located on the axis of the two cylindrical conductors.
-

Question (2)

- a) A small isolated conducting sphere of radius 2 cm is connected to a battery of 48 volt for a long time then the battery is disconnected. Surrounding this sphere and concentric with it a conducting spherical shell, which posses no net charge, is placed. The inner radius of the shell is 3 cm, and the outer radius is 5 cm. The first half of the space of the region I ($2 \text{ cm} < r < 3 \text{ cm}$) is filled with a dielectric permittivity $\epsilon_{r1} = 4$ while the second half of region I ($2 \text{ cm} < r < 3 \text{ cm}$) is filled with a dielectric permittivity $\epsilon_{r2} = 6$. The medium of region II ($r > 5 \text{ cm}$) is free space. If the outer surface of the shell is grounded (earthed), find:
- The capacitance of the system.
 - Electrical charge on each sphere.
 - Electric field intensity in all region,
 - The energy stored of the system.
- b) A potential in cylindrical coordinates is a function of r and ϕ but not in z . Obtain the separated differential equations for R and Φ and solve them. The region is charge-free.
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Question (3)

- a) Find the scalar magnetic potential in the region between the conductors of the coaxial cables with inner radius a and outer radii b and c . Then, find it at the point $(r, 50^\circ, \theta)$ where $a < r < b$.
- b) Explain the effect of applying an external magnetic flux density on a magnetic material. Sketch a microscopic delta volume of the magnetic material before and after applying the external magnetic field.
- c) The magnetic flux density in a magnetic material with $\chi_m = 9$ is given in a certain region as $\vec{B} = 0.01 y^2 \vec{a}_x \text{ T}$. At $y = 0.5 \text{ m}$ find the value of:
- i) \vec{M} ii) \vec{H} iii) \vec{J} iv) \vec{J}_m v) \vec{J}_{total} .
-

Question (4)

- a) Within a certain region, $\epsilon = 10^{-11} \text{ F/m}$ and $\mu = 10^{-5} \text{ H/m}$. If $B_x = 2 \times 10^{-4} \cos(10^5 t) \sin(10^{-3} y) \text{ T}$. Find:
- i) The electric field intensity,
ii) The total magnetic flux passing through the surface $x = 0, 0 < y < 40 \text{ m}, 0 < z < 2 \text{ m}$, at $t = 1 \mu\text{s}$,
ii) The EMF induced in the closed loop $x = 0, 0 < y < 40 \text{ m}, 0 < z < 2 \text{ m}$,
- b) A parallel plate capacitor 1 m square with 100 mm plate separation is filled with a medium for which $\sigma = 0.005 \Omega^{-1} / \text{m}$, $\epsilon_r = 20 - j10$, and $\mu_r = 1$. If 100 V (rms) at 10 MHz is applied. Assume a uniform field in the capacitor, find:
- i) Total current,
ii) The power factor and power loss as a heat,
iii) Equivalent circuit of the capacitor.
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Question (5)

- a) A wave propagating in a lossless dielectric has the component $\vec{E} = 200 \cos(10^5 t - \beta z) \vec{a}_x \text{ V/m}$ and $\vec{H} = 2 \cos(10^5 t - \beta z) \vec{a}_y \text{ A/m}$. If the wave is travelling at $v = 0.5 c$. Find (i) μ_r (ii) ϵ_r (iii) β (iv) λ (v) η .

- b) An electric field in a certain region is given by

$$\vec{E} = 50 e^{-0.001x} \sin(377 \times 10^9 t - 3770 x) \vec{a}_y \text{ V/m}$$

Find:

- i) The velocity of the wave travel,
ii) The wave impedance,
iii) The skin depth in the material in which the wave components are traveling,
iv) The Poynting vector.
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Question (6)

- a) A coaxial cable is terminated by an impedance equal to $50 + j100 \Omega$. The cable has a characteristic impedance equal to 50Ω and it is filled with dielectric, $\epsilon_r = 4$. The operating frequency is 5 GHz. Determine:
1. The normalized impedance,
 2. The propagation velocity,
 3. The propagation constant β ,
 4. The propagation wavelength λ ,
 5. The reflection coefficient at the load,
 6. The SWR,
 7. Minimum resistance in the transmission line and its positions in the cable,
 8. Maximum resistance in the transmission line and its positions in the cable,
- b) When a short circuit is placed at the load plane minima occur at the following positions: $z = 0.2 \text{ cm}$, 2.2 cm and 4.2 cm . The short circuit is replaced by the load, V_{\max} and V_{\min} are measured and the SWR is found to be 1.5. Voltage minimum occur at: $z = 0.72 \text{ cm}$, 2.72 cm , 4.72 cm . Determine the load.
- c) Starting from Maxwell's equations in Cartesian coordinates: Derive the relation between the transverse component of the electric, E_x , and the longitudinal components (E_z , H_z). Briefly explain the meaning of TEM, TE, TM, hybrid modes.
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Good Luck
